ICDEA 2017

23RD INTERNATIONAL CONFERENCE ON DIFFERENCE EQUATIONS AND APPLICATIONS

JULY 24 – JULY 28, TIMIŞOARA, ROMANIA

ABSTRACTS BOOK

International Society of Difference Equations

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ICDEA 2017

The 23rd International Conference on Difference Equations and Applications

ABSTRACTS BOOK

Timişoara, Romania
July 24-28, 2017

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PLENARY LECTURES

Global stability and the carrying simplex for discrete-time competitive population models

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Many discrete-time competitive population models feature a globally[1] attracting Lipschitz invariant manifold known as the carrying simplex (first appearing in [4, 3]), and this manifold has a strong influence on the global dynamics of the model. Apart from its pure aesthetic interest, the geometry of the carrying simplex can also inform on the model dynamics. In some well-known population models the computed carrying simplex appears to be convex or concave, although this seems fiendishly difficult to prove beyond planar models [1]. The geometry of the carrying simplex can be linked with global stability via the Split Lyapunov Method [2] originally developed for Lotka-Volterra differential equations [5].

Via some well-known competition models, and in particular the planar Leslie-Gower model [1], I will discuss carrying simplices and their geometry, then the Split Lyapunov Method, and finally how the two are linked.

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[1] All points in the first orthant except the origin

Discrete Delayed Exponential and its Applications (Representation of Solutions, Control Theory)

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With the use of a discrete matrix delayed exponential (which is a matrix formalism of the well-known step-method), new formulas are derived in [3, 4] for solutions of linear discrete systems with constant coefficients and single delay and used in [2] to solve a controllability problem. Recently, a growing interest in this topic has been observed. The original results of [2–4] have been generalized in several directions and, along with their generalizations, applied to various problems of the representation of solutions and control problems such as in [1, 5, 6]. The talk will give an overview of the previous and recent results adding some remarks on the possible avenues of future research.

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Towards a theory of global dynamics in difference equations: Application to population dynamics

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Global dynamics of difference equations/discrete dynamical systems are the most challenging problems in these disciplines. In this talk, we will explore some of the recent breakthroughs and advances in this area. The global dynamics of two types of discrete dynamical systems (maps) have been successfully established. These are triangular difference equations (maps) and monotone discrete dynamical systems (maps). We establish a general theory of triangular maps with minimal conditions. Smith's theory of planar monotone discrete dynamical systems is EXTENDED via a new geometric theory to any finite dimension. Then we show how to establish global stability for maps that are neither monotone nor triangular via singularity theory and the notion of critical curves.

Applications to models in biology and economics will be discussed.

Bifurcations in smooth and piecewise smooth noninvertible maps

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The last decades have been characterized by great achievements in the understanding of the dynamics of *smooth* systems, both in the regular and chaotic regimes. Characteristic features of *noninvertible* 2-dimensional maps have been studied by the French school on iteration theory dating back to the pioneering works of Gumowski and Mira ([3], [4]), as well as by their collaborators from other countries ([6]). Many properties may be generalized to n-dimensional maps, $n > 2$ (for example the homoclinic bifurcations of expanding fixed points or cycles [2], [8]). In this research field there are still many open problems which deserve to be investigated.

The interest towards the theory of *piecewise smooth maps* has recently been increased due to numerous studies in various applied fields, mainly in engineering and physics, leading to *nonsmooth* dynamical systems (see, e.g. [1], [7] and the survey [5]; for the one-dimensional case see [9], [10] and references therein). The study of local and global bifurcations of attractors and their basins of attraction, associated with nonsmooth systems will certainly be fruitful in the coming years.

In the present talk we shall recall some basic properties of smooth and nonsmooth noninvertible maps, outlining useful tools for investigation of their dynamics which may be of some help for future studies in this field.

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Non-autonomous dynamical systems: some surprising local and global results

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The study of periodic discrete dynamical systems is a classical topic that has attracted the researcher's interest in the last years, among other reasons, because they are good models for describing the dynamics of biological systems under periodic fluctuations whether due to external disturbances or effects of seasonality.

These k-periodic systems can be written as $x_{n+1} = f_{n+1}(x_n)$, with initial condition x_0, and a set of maps $\{f_m\}_{m\in\mathbb{N}}$ such that $f_m = f_\ell$ if $m \equiv \ell \,(\mathrm{mod}\ k)$ and can be studied via the *composition map* $f_{k,k-1,\ldots,1} = f_k \circ f_{k-1} \circ \cdots \circ f_1$.

The aim of this talk is to present some surprising phenomena, of local or global nature, appearing when we study them. For instance:

Theorem A. *(a) For all $n \geq 1$ there exist $k \geq 3$ polynomial maps $f_i : \mathcal{U} \subseteq \mathbb{R}^n \to \mathbb{R}^n$, for $i \in \{1, \ldots, k\}$, sharing a common fixed point p which is locally asymptotically for each map, and such that p is repeller for the composition map $f_{k,k-1,\ldots,1}$.*

(b) For all $n = 2m \geq 2$ there exist 2 polynomial maps $f_1, f_2 : \mathcal{U} \subseteq \mathbb{R}^{2m} \to \mathbb{R}^{2m}$, sharing a common fixed point p which is locally asymptotically for both maps, and such that p is repeller for the composition map $f_{2,1}$.

Recall, that the so called *Parrondo's paradox* is a paradox in game theory, that essentially says that *a combination of losing strategies becomes a winning strategy.* Theorem A can be interpreted as a kind of dynamical Parrondo's paradox.

Theorem B. *There exist two rational planar maps f_1 and f_2, well-defined in the first quadrant, such that f_1 has "complicated" dynamics, f_2 is rationally integrable and the composition maps $f_{2,1}$ and $f_{1,2}$ are rationally integrable.*

These maps are constructed studying the non-autonomous periodic second order Lyness difference equations $x_{n+2} = (a_n + x_{n+1})/x_n$, where $\{a_n\}$ is a cycle of k positive numbers, i.e. $a_{n+k} = a_n$.

The above results appear in the joint papers with Anna Cima and Víctor Mañosa, [1, 2].

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Computing areas of attraction and repulsion for nonautonomous, noninvertible dynamical systems

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Stable fiber bundles are the nonautonomous analog of stable manifolds and these objects provide valuable information on the underlying dynamics. We propose an algorithm for their approximation that is based on computing zero contours of particular operators. The resulting program applies to a wide class of models, including noninvertible and nonautonomous discrete time systems. Precise error estimates are provided and fiber bundles are computed for several examples. Extended results are presented that allow an approximation of the stable hierarchy of fiber bundles.

Finally, we apply the contour algorithm to (non)autonomous ODE models. For the famous three-dimensional Lorenz system, we calculate several approximations of the two-dimensional Lorenz manifold.

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Forward attractors and limit sets of nonautonomous difference equations

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The theory of nonautonomous dynamical systems has undergone major development during the past 19 years since I talked about attractors of nonautonomous difference equations at ICDEA Poznan in 1998.

Two types of attractors consisting of invariant families of sets have been defined for nonautonomous difference equations, one using pullback convergence with information about the system in the past and the other using forward convergence with information about the system in the future. In both cases, the component sets are constructed using a pullback argument within a positively invariant family of sets. The forward attractor so constructed also uses information about the past, which is very restrictive and not essential for determining future behaviour.

The forward asymptotic behaviour can also be described through the omega-limit set of the system.This set is closely related to what Vishik called the uniform attractor although it need not be invariant. It is shown to be asymptotically positively invariant and also, provided a future uniformity condition holds, also asymptotically positively invariant. Hence this omega-limit set provides useful information about the behaviour in current time during the approach to the future limit.

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Characterization of chaos for piecewise smooth maps

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Let f and g be piecewise smooth interval maps, with critical-singular sets, and A a cycle of intervals for f . We prove that A is a topological chaotic attractor if, and only if, A is a metric chaotic attractor. Let $h|A$ be a topological conjugacy between f and g. We prove that, if h is differentiable at a single point p of the visiting set V, with non zero derivative, then h is smooth in A. Furthermore, the visiting set V is a residual set of A and, if the sets C_f and C_g are critical then V has μ full measure, for every expanding measure μ, with supp $\mu = A$.

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Dynamics of the period maps of time periodic delay differential equations

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We give an overview of the dynamics generated by the period map of time periodic delay differential equations. First we discuss stability issues and Neimark-Sacker bifurcations that generate an invariant torus. We show that a common family of equations produce 1:4 strong resonances. For a class of linear equations with periodic coefficients satisfying a certain sign condition, we give a new and simply checkable sufficient and necessary condition for the stability of the zero solution. We construct specific examples to show that an intuitive criterion for the stability can easily fail, and explore stability switches for various ratios of the delay and the period. We show that any finite dimensional discrete dynamics can be realized by scalar periodic delay differential equations, and for an arbitrary map we explicitly construct the periodic equation such that its Poincaré-map generates a topologically equivalent dynamics to the n-dimensional dynamics generated by the iterates of the original map. Finally, we show how these results can be used to explain some real world phenomena, including malaria dynamics in temperate regions, irregular cholera outbreaks or the El-Nino phenomenon in climatology.

Persistence versus extinction for a class of discrete-time structured population models

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We provide sharp conditions distinguishing persistence and extinction for a class of discrete-time dynamical systems on the positive cone of an ordered Banach space generated by a map which is the sum of a positive linear contraction A and a nonlinear perturbation G that is compact and differentiable at zero in the direction of the cone. Such maps arise as year-to-year projections of population age, stage, or size-structure distributions in population biology where typically A has to do with survival and individual development and G captures the effects of reproduction. The threshold distinguishing persistence and extinction is the principal eigenvalue of $(I - A) - 1G'(0)$ provided by the Krein-Rutman Theorem, and persistence is described in terms of associated eigen-functionals. Our results are illustrated by application to a plant model with a seed bank.

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PRESENTATIONS

On modular happy numbers

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In this paper, we investigate the asymptotic behavior of the sequences generated by iterating the process of summing the modular powers of the decimal digits of a number. In particular, we identify all *modular happy numbers*. A number is called modular happy if the sequence obtained by iterating the process of summing the modular powers of the decimal digits of the number ends with 1.

Statistical approximation and q-generalizations of some linear processes

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This talk is focused on sequences of linear positive operators, the starting point being represented by Popoviciu-Bohman-Korovkin criterion. Our first aim is to sum up recent investigation on statistical convergence of this type of approximation processes. The second aim is to construct a bivariate extension of Stancu discrete operators. This generalization is based on q-integers and on the tensor product method.

Mathematical Modelling of the Growth of Pseudomonas putita by Gompertz Dynamic Equations

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The main importance of Gompertz function is able to be applicable to different types of growth models like tumor, human fetus, human life. In this study, we propose two Gompertz dynamic equations in order to observe the best fit with the measurement results of the growth of the Pseudomonas putida. The variation of parameters on time scales is our approach to show the existence and uniqueness of the solutions of initial value problems of Gompertz dynamic equations. We also use mathematica codes the best fitting to the data which are taken from [1].

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Prices under differentiation

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We develop a theoretical framework to study the location-price competition in a Hotelling-type network game, extending the Hotelling model with linear transportation costs from a line (city) to a network (town). We show the existence of a pure Nash equilibrium price if, and only if, some explicit conditions on the production costs and on the network structure hold. Furthermore, we prove that the local optimal localization of the firms are at the cross-roads of the town.

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Infinite dimensional discrete operators

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This work is devoted to infinite dimensional discrete operators that can be considered as a difference analog of differential equations on the whole axis. We obtain a necessary and sufficient condition in order for the linear operator to be normally solvable. Topological degree for nonlinear operators is constructed. insert here the text of the abstract.

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Nonlinear Dynamics of fractional order Model of CD4$^+$ T Cells

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In this paper we studied the cycling CD4$^+$ T cells model of HIV infection of arbitrary order. We consider logistic tumor growth because in infected patients the division rate of CD4$^+$ T cells deceases approximately linearly with the CD4 T cell count, suggesting that the growth rate is density dependent and is governed by a logistic-like growth function. A variety of clinical data sets suggest that virus replication is limited by the availability of CD4$^+$ T cells. Therefore we investigate the dynamics of our model by finding equilibrium points of our model. An implicit numerical scheme using finite difference approximation is proposed which shows an excellent degree of accuracy when applied to fractional HIV model. A comparison is made between numerical simulations of proposed implicit numerical scheme and Grunwald-Letnikov method; we conclude that proposed scheme is reliable to obtain realistic positive numerical solutions of the HIV model. We present the numerical simulations of the HIV model to illustrate the dynamics for different fractional order.

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On the dynamical properties of certain complex recurrent sequences

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Having been investigated for centuries, recurrent sequences are still relevant for mathematics and sciences. For example, Fibonacci numbers found applications in arts and sciences, and still inspire numerous algorithms in computer science.

Generalizing Fibonacci numbers to the complex plane, a Horadam sequence $\{w_n\}_{n=0}^{\infty}$ is defined by the recurrence $w_{n+2} = pw_{n+1} + qw_n$, $w_0 = a, w_1 = b$, where the parameters a, b, p and q are complex numbers.

The geometry and dynamics of complex Horadam sequences and generalizations have been explored extensively over the past few years. Periodic Horadam sequences have been characterized in [1], while their geometric structure was investigated in [5]. A wide variety of non-periodic Horadam orbits (stable, quasi-convergent, convergent or divergent) was presented in [4], some of inspiring the design of a pseudo-random number generator, investigated by Bagdasar and Chen [2].

This study aims to further explore the dynamics of Horadam orbits and their generalizations. First, certain extensions of the results concerning the periodicity of perturbed Horadam sequences given in [6] (where some constant and periodic perturbations were considered) are presented. Then, we investigate the dynamics of generalized complex Horadam sequences of arbitrary order (whose periodicity was investigated in [3]) in a more general context. Finally, links to the trichotomy concepts for linear discrete-time systems discussed in [7] will be formulated.

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Global Stability of Periodic Monotone Maps

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In a recent article, Balreira, Elaydi and Luís [1] have developed a geometric approach to monotonicity for maps defined on Euclidean spaces $\mathbb{R}_+^k$, of arbitrary dimension k. They have shown that if the inverse of the Jacobian matrix is positive, then under analytical hypotheses that are motivated from applications to mathematical biology and economics, one can show that Kolmogorov monotone maps on $\mathbb{R}_+^k$ have a globally asymptotically stable fixed point.

In this talk we will will investigate the extension of these results to the global stability of periodic monotone maps. The new concept of monotonicity is invariant under composition of maps and, surprisingly, the verification of the analytical hypotheses became a question of global injectivity of certain maps associated to the periodic system. Namely, given a Kolmogorov type map $F(\mathbf{x}) = (x_1 f_1(\mathbf{x}), x_2 f_2(\mathbf{x}), \ldots, x_k f_k(\mathbf{x}))$, we say that the associated *reduced* Kolmogorov map is given by $\widetilde{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x}))$. Using classical results for global injectivity, we establish conditions under which the periodic system is globally stable. We illustrate our techniques by analyzing the Ricker competition model.

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Derivatives on time scales relative to the class of the controlled (or conformable) derivatives

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In the 1980s, S. Hilger has defined the derivatives on time scales, to unify discrete and continuous dynamics [1]. Such definition refers to the usual notion of derivatives in Calculus. The fractional (or non-integer) derivatives has its beginning in the XVII century [4]. In the last few years the notion of controlled (or conformable) derivatives emerged in the literature [2]. Nowadays it is discussed whether such derivatives can be considered properly as of fractional kind or not [5, 3]. In this work, we are being defining the derivatives on time scales relative to the class of the controlled general derivatives, and will be giving some basic properties on them.

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Two asymptotically different positive solutions of a perturbed discrete equation

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Denote $Z_s^q := \{s, s+1, \dots, q\}$ where s and q are integers such that $s \leq q$. Similarly a set Z_s^∞ is defined. The paper considers the scalar linear discrete equation with delay

$$\Delta x(n) = -\left(\frac{k}{k+1}\right)^k \frac{1}{k+1} x(n-k) + \omega(n) \tag{1}$$

where a perturbation function $\omega\colon Z_a^\infty \to R$ will be more precisely defined in the talk, $k \geq 1$ is a fixed integer, $n \in Z_a^\infty$, and a is a whole number.

We prove that there exist two positive solutions $x = x_1(n)$, $x = x_2(n)$ of equation (1) defined on $n \in Z_a^\infty$ such that $x_1(n)$ does not depend linearly on $x_2(n)$. Moreover,

$$\lim_{n\to+\infty} \frac{x_2(n)}{x_1(n)} = 0.$$

The boundaries of perturbations guaranteeing the existence of a positive solution or a bounded vanishing solution of a perturbed linear discrete delayed equation are given. The proofs are based on a discrete variant of Ważewski's topological method [1] and motivated by the method of asymptotic decompositions [2].

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A Perron type approach for individual exponential dichotomy for linear skew-product semiflows

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The theory of linear skew-product semiflows emerged from the desire to unify the study of asymptotic behavior for evolutionary families. In this situation, in order to obtain uniform asymptotic behavior of the evolutionary families, in literature is considered mostly the case in which the constant K from the Boundedness Theorem is independent of $\theta \in \Theta$.

This paper addresses, using the Perron approach, the problem of exponential dichotomy for a cocyle $\Phi(\theta, \cdot)$ over a continuous semiflow σ, for every $\theta \in \Theta$ fixed, by renouncing to the possibility to choose the constant K from the Boundedness Theorem as independent of $\theta \in \Theta$.

The main theorem proves that, for each $\theta \in \Theta$, the linear skew-product semiflows have a similar asymptotic behavior like the behavior of the evolutionary processes which is highlighted by V. Minh, which is so-called individual exponential dichotomic behavior for linear skew-product semiflows.

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On extensions of 1D Drossel-Schwabl forest-fire model

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We reformulate 1D forest-fire model by Drossel and Schwabl in terms of energy accumulation and release in order to define Random Domino Automaton - a stochastic cellular automaton being a toy model of earthquakes. We define respective parameters related to probability of energy accumulation and release depending on its amount and propose a set of discrete equations describing stationary state of the automaton.

We demonstrate that for some class of parameters the distribution of accumulated energy is related to Motzkin numbers and present also a construction of similar cellular automaton related to Catalan numbers.

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On uniform exponential splitting of variational nonautonomous difference equations in Banach spaces

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In this paper we study a concept of uniform exponential splitting, as a generalization of uniform exponential dichotomy for a cocycle C over a semiflow S.

Discrete characterizations of this concept in terms of Datko's type, respectively Lyapunov functions are obtained from the point of view of the projector families (invariant and strongly invariant).

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Discrete version of an optimal partitioning problem

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Recently a great interest has been shown towards problems regarding partitions $(\omega_i)_{i=1..n}$ of a domain D, a subset an euclidean space or of a manifold, which minimize some spectral functionals depending on the spectrum of the Dirichlet Laplace operator. The main examples are

$$\mathcal{F}(\omega_i) = \lambda_1(\omega_1) + ... + \lambda_1(\omega_n) \quad \text{and} \quad \mathcal{F}(\omega_i) = \max_{i=1..n} \lambda_1(\omega_i).$$

Due to the complexity of the problem, explicit solutions are rarely known, thus it is important to be able to produce numerical simulations which approximate optimal partitions. In order to do this we consider a discrete approximation of the optimal partitioning problem. We present some properties of the discrete problem and describe how to produce an efficient implementation. The algorithm allows us to find numerical optimal partitions for a large class of domains D in $\mathbb{R}^2, \mathbb{R}^3$ and on surfaces in $\mathbb{R}^3$. The numerical algorithm is inspired by [1] and more details about the contents of this talk can be found in [2].

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Higher-Order Beverton–Holt Equations

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This is joint work with Fozi Dannan and Sabrina Streipert. In this talk, we discuss a certain nonautonomous Beverton–Holt equation of higher order. After an introduction to the classical Beverton–Holt equation and recent results, we solve the higher-order Beverton–Holt equation by rewriting the recurrence relation as a difference system of order one. In this process, we examine the existence and uniqueness of a periodic solution and its global attractivity. We continue our analysis by studying the corresponding second Cushing–Henson conjecture, i.e., by relating the average of the unique periodic solution to the average of the carrying capacity.

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On polynomial asymptotic behaviors for discrete-time systems in Banach spaces

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The aim of the paper is to study polynomial asymptotic properties for linear discrete-time systems in Banach spaces. We deal with different polynomial concepts and we give some representative theorems related to this topic.

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Stability of two dimensional incommensurate fractional-order systems

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This paper presents necessary and sufficient conditions for the asymptotic stability and instability of the null solution for two-dimensional autonomous linear incommensurate fractional-order dynamical systems with Caputo derivatives of different orders, of the form

$$\begin{cases} {}^cD^{q_1}x(t) = a_{11}x(t) + a_{12}y(t) \\ {}^cD^{q_2}x(t) = a_{21}x(t) + a_{22}y(t) \end{cases}$$

where $A = (a_{ij})$ is a real two-dimensional matrix and $q_1, q_2 \in (0, 1)$ are the fractional orders of the Caputo derivatives.

The theoretical results are obtained employing Laplace transforms and their asymptotic expansions, as well as complex analysis tools, leading to a generalization of the well-known Routh-Hurwitz criterion.

As an application, the obtained theoretical findings are later used for investigating the stability properties of a two-dimensional fractional-order conductance-based neuronal model:

$$\begin{cases} {}^cD^{q_1}v(t) = I - I(v, w) \\ {}^cD^{q_2}w(t) = \phi(w_\infty(v) - w) \end{cases}$$

where v represents the membrane potential and w is a recovery variable of the biological neuron.

Moreover, the occurrence of Hopf bifurcations is also discussed, choosing the fractional orders q_1, q_2 as bifurcation parameters. Numerical simulations are also presented to illustrate the theoretical results.

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Structured stability radii and exponential stability tests for Volterra difference systems

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Uniform exponential (UE) stability of linear difference equations with infinite delay is studied using the notions of a stability radius and a phase space. The state space $\mathcal{X}$ is an abstract Banach space. We work with non-fading phase spaces $c_0(\mathbb{Z}^-, \mathcal{X})$ and $\ell^\infty(\mathbb{Z}^-, \mathcal{X})$ and with exponentially fading phase spaces of the ℓ^p and c_0 types. For equations of the convolution type, several criteria of UE stability are obtained in terms of the Z-transform $\widehat{K}(\zeta)$ of the convolution kernel $K(\cdot)$, in terms of the input-state operator and of the resolvent (fundamental) matrix. These criteria do not impose additional positivity or compactness assumptions on coefficients $K(j)$. Time-varying (non-convolution) difference equations are studied via structured UE stability radii $\mathrm{r_t}$ of convolution equations. These radii correspond to a feedback scheme with delayed output and time-varying disturbances. We also consider stability radii $\mathrm{r_c}$ associated with a time-invariant disturbance operator, unstructured stability radii, and stability radii corresponding to delayed feedback. For all these types of stability radii two-sided estimates are obtained. The estimates from above are given in terms of the Z-transform $\widehat{K}(\zeta)$, the estimate from below via the norm of the input-output operator. These estimates turn into explicit formulae if the state space $\mathcal{X}$ is Hilbert or if disturbances are time-invariant. The results on stability radii are applied to obtain various exponential stability tests for non-convolution equations. Several examples are provided.

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On the equivalence of the Melnikov functions method and the averaging method

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The Melnikov functions method and the averaging method are both tools for finding limit cycles of analytic planar differential systems which are perturbations of a period annulus. In order to apply the averaging method, one needs to consider some change of variables to transform the planar system into a scalar periodic equation which perturbs a continuum of constant solutions. In this talk we present results obtained in [1], where we proved the equivalence of these two methods with respect to any possible change of variables. More precisely, we proved that the Poincaré return map of the planar system and the Poincaré translation map of the scalar equation coincide. For distinct specific changes of variables this was stated before by Buică–Llibre [2] and proved by Han–Romanovski–Zhang [3].

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Some Properties of Difference Equations with Infinitely Many Cycles with Period Two

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We consider such difference equations for which exist infinitely many cycles with period two. In these cases we have observed that the characteristic equation of the linearized equation of considered difference equation always has a root -1. In the article [1] we investigated three second-order rational difference equations with period-two solutions and we proved that the characteristic equation of the linearized equation of these difference equations has a root -1. Also in all three cases the points of the cycle are located on a hyperbola. In the book [2] authors mention the similar observation about dynamics of some third-order rational difference equations (page 107) and that every solution of such difference equations converges to a period-two solution.

What is the relationship between these three subjects (root -1, infinitely many cycles with period two, convergence of solutions)? We discuss this connection in the presentation.

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Almost reducibility of linear difference systems from a spectral point of view

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We prove that, under some conditions, a linear nonautonomous difference system is Bylov's almost reducible to a diagonal one whose terms are contained in the Sacker and Sell spectrum of the original system.

In the above context, we provide an example of the concept of diagonally significant system, recently introduced by Pötzsche. This example plays an essential role in the demonstration of our results.

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On a discrete-time model with replicator dynamics in renewable resource exploitation

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In the recent paper [1] is considered a discrete time version of the model proposed by [2] which aims to describe a fishery where a population regulated by a logistic growth function is exploited by a pool of agents that can choose, at each time period, between two different harvesting strategies according to a profit-driven evolutionary selection rule. The resulting discrete dynamical system, represented by a two-dimensional nonlinear map, is characterized by the presence of invariant lines on which the dynamics are governed by one-dimensional restrictions that represent pure strategies. Interesting dynamics related to interior attractors, where players playing both strategies coexist, are evidenced by analytical as well as numerical methods that reveal local and global bifurcations. In particular, it is shown that under the assumption of a logistic growth model for the fish stock, chaotic dynamics can occur along the pure strategy invariant manifolds, so that chaos synchronization as well as bubbling and blowout phenomena are observed, thus revealing the existence of Milnor attractors. Generally speaking, those attractors have interesting economic implications not only for mentioned example, but also for the entire class of evolutionary games that this model could represent. In the present work we start studying this class of evolutionary games based on the analysis of the non-topological Milnor attractors and related riddled basins.

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Sturm-Liouville abstract problems for second order differential equations in non commutative case

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In this work, we prove some new results on Sturm-Liouville abstract problems of second order differential equations of elliptic type in a non-commutative framework. We study the case where the second member of the differential-operator equation belong to $L^p(0,1;X)$, with general $p \in]1,+\infty[$ and X being a UMD Banach space. Existence, uniqueness and optimal regularity of the strict solution are proved. This work improves naturally the ones studied by Cheggag, Favini, Labbas, Maingot and Medeghri in the commutative case, see [1], [2] and [3].

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On (h,k)-dichotomy of linear discrete-time systems in Banach spaces

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The paper considers two concepts of dichotomy with different growth rates for linear discrete-time systems in Banach spaces. Characterizations (in terms of Lyapunov type norms) and connections between these concepts are given. The approach is motivated by various examples.

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The hypergeometric difference equation

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Generalized hypergeometric series are classical special functions that unify many common special functions into one. We present a discrete analogue of the generalized hypergeometric functions that unifies many well-known discrete special functions into one.

Time scales derivatives and the Radon transform in the plane

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In this work, we will be considering the derivative on time scales in two variables on an arbitrary nonempty set in the plane. It will be considered the Radon transform and properties concerning the replacement of images in this environment.

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Existence and Asymptotic Behavior of Positive Solutions for a Fractional Semilinear Dirichlet Problem

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We consider the following semilinear fractional initial value problem

$$D^{\alpha}u(x) = a(x)u^{\sigma}(x), \quad x \in (0,1) \text{ and } \lim_{x \longrightarrow 0^+} x^{1-\alpha}u(x) = 0,$$

where $0 < \alpha < 1$, $\sigma < 1$ and a is a positive measurable function on $(0,1)$.

We establish the existence and the uniqueness of a positive solution in the space of weighted continuous functions. We also give the boundary behavior of such solution.

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Global stability in difference equations

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We consider 2- and 3-dimensional maps depending on a parameter. Local stability of a fixed point is known up to a critical parameter value where Neimark-Sacker bifurcation takes place. The aim is to show global stability for all parameter values where local stability holds. Near the fixed point analytical tools are used to construct a neighbourhood $\mathcal{N}$ belonging to the domain of attraction of the fixed point. The size of the neighbourhood $\mathcal{N}$ is crucial since outside $\mathcal{N}$ rigorous, computer-aided calculations are applied to show that each point enters into $\mathcal{N}$ after finite number of iterations. The 3-dimensional case is technically more complicated as it requires a center manifold reduction, and in particular, an estimation for the size of the center manifold is important. Among others, the difference equations

$$x_{k+1} = ax_k(1 - x_{k-d})$$

and

$$x_{k+1} = x_k e^{a - x_{k-d}}$$

where a is a positive parameter and $d = 1$ or $d = 2$, can be handled with our technique.

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Stability of zero solutions of linear differential equations with random coefficients and random transformation solution

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We consider the linear differential equations with random Semi-Markov coefficients [1]. For definition that systems we introduce a system non-random n equations which has solutions in every realization Semi-Markov process and find the fundamental matrix - solutions of this usually differential equations systems. We assume that in some time solutions of initial system have random transformations. We introduce Lyapunov function and introduce new definition L2-stability: Zero solution of the systems with random coefficients with random transformation of solution - L2-stable, if convergence of integral from 2- mean of random solutions. Our problems: constructing a system of equations for Lyapunov function in any realization of Semi-Markov process. We proved Theorem: If for corresponding determinant system exists the Lyapunov function, then zero solution of initial system with random coefficients is L2- stable.

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Towards bifurcation theory of continuous and discrete polynomial dynamical systems

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We carry out the global bifurcation analysis of continuous and discrete polynomial dynamical systems [1]. First, using new bifurcational and topological methods, we solve *Hilbert's Sixteenth Problem* on the maximum number of limit cycles and their distribution for the 2D general Liénard polynomial system [2], Holling-type quartic dynamical system [3], and Kukles cubic system [4]. Then, applying a similar approach, we study 3D polynomial systems and complete the strange attractor bifurcation scenario for the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles which is related to *Smale's Fourteenth Problem* [5]. We discuss also how to apply our approach for studying global limit cycle bifurcations of discrete polynomial dynamical systems which model the population dynamics in biomedical and ecological systems.

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Morse Decomposition for Scalar Delay Difference Equations

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Consider the following class of difference equations

$$x_{k+1} = f(x_k, x_{k-d}),$$

where $f\colon \mathbb{R}^2 \to \mathbb{R}$ is a C^1 function, which is strictly increasing in its first variable and fulfils either $yf(0,y) > 0$ or $yf(0,y) < 0$ for all $y \in \mathbb{R} \setminus \{0\}$. Under the assumption that the global attractor exists, we give a Morse decomposition of it. This gives some insight into the structure of the global attractor. The decomposition is based on an integer valued Lyapunov function introduced by J. Mallet-Paret and G. Sell [2] as a discrete time counterpart of their celebrated discrete Lyapunov function for delay differential equations [1]. Our results apply e.g. to several discrete-time models arising from life-sciences, such as May's genotype model, the Wazewska–Lasota equation or the discrete Mackey–Glass equation.

This is a joint work with Christian Pötzsche (Alpen-Adria Universität Klagenfurt).

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Iterated function systems consisting of φ-max-contractions have attractor

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We associate to each iterated function system consisting of φ-max-contractions an operator (on the space of continuous functions from the shift space on the metric space corresponding to the system) having a unique fixed point whose image turns out to be the attractor of the system. Moreover, we prove that the unique fixed point of the operator associated to an iterated function system consisting of convex contractions is the canonical projection from the shift space on the attractor of the system.

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Series solutions of dynamic equations on time scales

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In this talk, a series solutions method for dynamic equations of arbitrary order on time scales is discussed. The method generalizes the results presented in [3], [4] and [5] which can be regarded as particular cases of this study. The most important feature of the method is that it can be applied to any time scales with constant or variable graininess function. As specific examples, solutions of Hermite and Legendre dynamic equations on various time scales are obtained by the series solution method.

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Multidimensional Brownian motion on time scales

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The theory of probability on time scales is in its inception. This work is devoted to develop the probability theory and introduce the n-dimensional Brownian motion on general time scales. The main issue in carrying out this construction is to generalize the well-known concepts such as probability space, probability density function, normal distribution on time scales. We first introduce the fundamentals of probability theory on time scales such as probability measure derived from Lebesgue Δ-measure, probability density function and generalized normal distribution. Then, the stochastic processes on general time scales will be defined. This definition indeed unifies the stochastic processes theory in discrete and continuous times. We conclude our talk by introducing n-dimensional Brownian motion on general time scales.

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Positive radial solutions for systems involving potential Lane-Emden nonlinearities

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Using topological degree arguments, critical point theory and lower and upper solutions method we establish non-existence, existence and multiplicity of positive radial solutions for one parameter systems involving potential Lane-Emden nonlinearities,

$$\begin{cases} \mathcal{M}(\mathrm{u}) + \lambda\mu(|x|)(p+1)\mathrm{u}^p\mathrm{v}^{q+1} = 0, & \text{in } \mathcal{B}(R), \\ \mathcal{M}(\mathrm{v}) + \lambda\mu(|x|)(q+1)\mathrm{u}^{p+1}\mathrm{v}^q = 0, & \text{in } \mathcal{B}(R), \\ \mathrm{u}|_{\partial\mathcal{B}(R)} = 0 = \mathrm{v}|_{\partial\mathcal{B}(R)}. \end{cases}$$

Here, $\mathcal{B}(R) = \{x \in \mathbb{R}^N : |x| < R\}$, $N \geq 2$ is an integer, $\mu : [0, R] \to [0, \infty)$ is continuous, $\mu > 0$ on $(0, R]$, the exponents p, q are positive, with $\max\{p, q\} > 1$ and $\mathcal{M}$ stands for the mean curvature operator in Minkowski space

$$\mathcal{M}(\mathrm{w}) = \operatorname{div}\left(\frac{\nabla \mathrm{w}}{\sqrt{1 - |\nabla \mathrm{w}|^2}}\right).$$

This talk is based on joint work with Petru Jebelean [1].

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Global and Local Stability Analysis in a Discrete-time Ramsey Model

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The choice of time as a discrete or continuous variable may radically affect the stability of equilibrium in an endogenous growth model with durable consumption. In the continuous-time model the steady state is locally saddle-path stable with monotonic convergence. However, in the discrete-time model the steady state may be unstable or saddle-path stable with monotonic or oscillatory convergence.

In this paper, we study general polynomial discretization in backward and forward looking, and the preservation of stability properties. We apply these results to the Ramsey model. Finally, in this paper, we study the local and global dynamics of a new discrete Ramsey model.

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Asymptotic Behavior of a Competitive System of Third-Order Difference Equations

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The purpose of this talk is to study the dynamical behavior of positive solutions for a system of rational difference equations of the following form

$$u_{n+1}=\frac{\alpha u_{n-1}}{\beta+\gamma v_n^p v_{n-2}^q},\ v_{n+1}=\frac{\alpha_1 v_{n-1}}{\beta_1+\gamma_1 u_n^p u_{n-2}^q},\ n=0,1,...$$

where the parameters $\alpha,\beta,\gamma,\alpha_1,\beta_1,\gamma_1,p,q$ are positive real numbers and the initial values u_{-i},v_{-i} are non-negative real numbers for $i=0,1,2$.

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Dynamics of the three-dimensional Ricker model

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In this talk I consider the discrete-time Ricker model of n competing types. I start by a derivation of the model and show that under mild condition the system admits a carrying simplex, that is, an attracting invariant hypersurface of codimension one. I then concentrate on the case $n = 3$ and show that there are essentially 33 different types of dynamics on the carrying simplex. In particular, I consider the question of which dimorphic population could have arisen by natural selection and study invasion of a third type into a dimorphic population.

Existence and stability of almost periodic solutions in discrete almost periodic systems

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In this talk, we shall consider the linear almost periodic system with variable coefficients

$$x_{n+1} = A(n)x_n, \;\; x_n \in R^n, \;\; n \geq n_0 \geq 0, \tag{1}$$

and their applicastion for some type of nonlinear system (cf. [1,3]). Even in nonlinear case, system (1) plays an important role, as their variational equations and moreover, it is requested to determine the (uniformly) asymptotical stability of the zero solution from the condition about $A(n)$. In the case where $A(n)$ is a constant matrix, it is well known that the stability is equivalent to the following condition (cf. [2]); "Absolute values of all eigen values of $A(n)$ are less than one."
However, it is not true in the case of variable coefficients, and hence we need additional conditions to (1). In the main theorem, we shall show that one of the such conditions is the diagonal dominance matrix condition on $A(n)$ [3]. This result improve a stability criterion based on results of R. D. Jenks [4] and F. Nakajima [5] for differential equations.

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Conditional oscillation of second order linear and half-linear difference and differential equations

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The talk is based mainly on the joint works [1, 2, 3] with Michal Veselý. The main subject of this talk is to present the results concerning the conditional oscillation of second order linear and half-linear difference and differential equations. We show the conditions which are sufficient for studied equations to be conditionally oscillatory, i.e., that there exists a border value given by their coefficients which separates oscillatory equations from non-oscillatory ones. We explicitly determine this borderline for the considered equations, including perturbed differential equations. We also discuss the so-called critical case which is solvable in general only for some special types of equations.

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From random dynamics to partially hyperbolic dynamics

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I'll discuss iterated function systems of diffeomorphisms on compact manifolds with a focus on synchronization by noise and on-off intermittency. The theory will be connected to the seemingly different context of Fubini's nightmare in stably ergodic diffeomorphisms.

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Fractional order integro-differential equations solution by artificial neural networks approach

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Great care must be taken in considering the fact that neural networks moved in the direction of a systematic world such as applied mathematics and engineering sciences. Such certain movement helped shaping fantastic changes in the numerical solution of complicated cases which are overt in natural phenomena. In the present study, a comprehensive optimization mechanism consisting of a reliable three-layered feed-forward neural network is formed to solve a class of fractional order ordinary integro-differential equations. One point should be kept in mind that the supervised back-propagation type learning algorithm which is based on the gradient descent method, is capable of approximating the mentioned problem on an arbitrary interval to any desired degree of accuracy. Besides, some comparative test problems are given to reveal the flexibility and efficiency of the proposed method.

Global Dynamics of Monotone Second Order Difference Equation

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We investigate the global character of the difference equation of the form

$$x_{n+1} = f(x_n, x_{n-1}), \quad n = 0, 1, \ldots$$

with several period-two solutions, where f is decreasing in the first variable and is increasing in the second variable. We show that the boundaries of the basins of attractions of different locally asymptotically stable equilibrium solutions or period-two solutions are in fact the global stable manifolds of neighboring saddle or non-hyperbolic equilibrium solutions or period-two solutions. We illustrate our results with many applications.

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The Statement of Polynomial Pencil of Sturm-Liouville Operators on Continuous and Discrete case

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Let us consider Polynomial pencil of Sturm-Liouville problem

$$-y^{''}+\left[q_0(x)+\lambda q_1(x)+...+\lambda_{n-1}q_{n-1}(x)\right]y=\lambda^{2n}y \tag{1}$$

$$y(0)=y(\pi)=0, \tag{2}$$

where λ is a real parameter, $y=y(x,\lambda)$ is unknown function and $q_k(x)$, $\overline{k=0,n-1}$ are continuous real-valued functions on $[0,\pi]$. In this study, inverse problem for (1), (2) investigated. By using Prüfer substitution, we obtained asymptotics of spectral parameters and a reconstruction formula for all the functions $q_k(x)$.

Similiar results for discrete case of (1), (2) will be interesting.

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Global Behavior of Some Nonlinear Nonautonomous Difference Equations

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In this paper we study the global asymptotic behavior of positive solutions of the nonlinear nonautonomous difference equation of the form

$$x_{n+1} = a_n x_n f(x_{n-k}), \qquad n = 0, 1, ...$$

with initial conditions

$$x_{-k}, ..., x_{-1} \geq 0, \quad x_0 > 0$$

where the following hypotheses are satisfied:

($\mathbf{H}_1$) The sequence $\{a_n\}$ is positive and periodic with period p, that is

$$a_{n+p} = a_n, \quad n = 0, 1, ...$$

($\mathbf{H}_2$) The function $f \in C[[0, \infty), [0, \infty)]$ is decreasing on $[0, \infty)$.

($\mathbf{H}_3$) The function $xf(x)$ is increasing on $[0, \infty)$.

($\mathbf{H}_4$) The function $f(tx)/f(x)$ is increasing on $[0, \infty)$ for $t \in (0, 1)$ and decresing on $[0, \infty)$ for $t \in (1, \infty)$.

We studied the permanence, extreme stability, periodicity, and oscillations including the character of semicycles. The obtained results are applied to several classic periodically forced population models including Beverton-Holt (and equivalent Pielou logistic), Ricker, and Maynard-Smith, models.

On the Rational Difference Equation with Quadratic Term

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We give the boundedness character, local and global stability of solutions of the following second-order rational difference equation with quadratic denominator,

$$x_{n+1} = \frac{\alpha + \gamma x_{n-1}}{Bx_n + Dx_n x_{n-1} + x_{n-1}}$$

where the coefficients are positive real numbers and the initial conditions, x_{-1}, x_0, are nonnegative real numbers such that the denominator is defined.

On the integrable deformations of a Hamilton-Poisson system

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A method to construct integrable deformations of a three-dimensional Hamilton-Poisson system is used in the case of an integrable version of the Rikitake system. Considering some particular functions of deformation a Hamilton-Poisson system is obtained. A study of this system from standard and nonstandard Poisson geometry points of view is performed, namely the stability of the equilibrium points and the existence of periodic orbits are analyzed. Furthermore, an explicitly construction of the homoclinic and heteroclinic orbits is presented. In addition, some connections between the energy-Casimir mapping associated to the considered system and the above-mentioned dynamical elements are pointed out.

A joint work with Ciprian Hedrea and Camelia Petrişor from *Department of Mathematics, Politehnica University of Timişoara, Romania.*

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A complete characterization of exponential stability for discrete dynamics

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For a nonautonomous dynamics defined by a sequence of bounded and possibly not invertible linear operators, we give a complete characterization of exponential stability in terms of invertibility of a certain operator acting on suitable Banach sequence spaces. We connect the invertibility of this operator to the existence of a particular type of admissible exponents. For the bounded orbits, exponential stability results from a spectral property. Some adequate examples are presented to emphasize some significant qualitative differences between uniform and nonuniform behavior.

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Generating functions for the number of paths on multidimensional integer lattice

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In a problem of generalized Dyck paths (see [1]) and vector partition functions (see [2]) assosiated to $\triangle$ we are given a finite set of steps $\triangle = \{\alpha^1, \alpha^2, \ldots, \alpha^N\}$, lying in the open half-space, where $\alpha^j = (\alpha_1^j, \alpha_2^j, \ldots, \alpha_n^j), j = 1, \ldots, N$ and α_i^j is an integer number. A number $g(y)$ of paths going from the origin to the point $y \in \mathbb{Z}^n$ using the steps from $\triangle$ is described by a difference equation $g(y) = \sum\limits_{i=1}^{N} g(y - \alpha^i)$. Let π_j be an operator such that $\pi_j Q(z_1, \ldots, z_n) = Q(z_1, \ldots, z_{j-1}, 0, z_{j+1}, \ldots, z_n)$, J be a set $\{j_1, \ldots, j_k\}$, where $1 \leqslant j_1 < j_2 < \ldots < j_k \leqslant n$, and $\pi_J = \pi_{j_1} \circ \pi_{j_2} \circ \ldots \circ \pi_{j_k}$.

Theorem. If $g(y)$ is a solution to a difference equation $g(y) = \sum\limits_{i=1}^{N} g(y - \alpha^i)$, then the generating function $F(z)$ of the sequence $g(y)$ satisfies the formula $\sum\limits_{J} (-1)^{\#J} \pi_j \, Q(z) F(z) = 0$, where $Q(z) = 1 - z_1^{\alpha_1^1} z_2^{\alpha_2^1} \cdot \ldots \cdot z_n^{\alpha_n^1} - \ldots - z_1^{\alpha_1^N} z_2^{\alpha_2^N} \cdot \ldots \cdot z_n^{\alpha_n^N}$ and $\#J$ is a number of elements of J.

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This work of second author was financed by the PhD SibFU grant for support of scientific research No: 14.

A Class of Anti-Competitive Systems of Difference Equations

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There are 112 systems of two rational-linear difference equations where, for all positive values of the parameters, the corresponding map is anticompetitive and the square of the map is strongly competitive. We present a theorem that describes the global behavior of the solutions for all 112 systems.

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New nonlinear estimates for surfaces in terms of their fundamental forms

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We establish several estimates of the distance between two surfaces immersed in the three-dimensional Euclidean space in terms of the distance between their three fundamental forms, measured in various Sobolev norms (see [2]). By imposing appropriate additional geometrical assumptions, we show that the dependence of the third fundamental form can be avoided. These estimates generalize in particular the nonlinear Korn inequality established by P.G. Ciarlet, L. Gratie and C. Mardare [1].

We also show how these nonlinear Korn inequalities can be applied to the nonlinear Koiter shell model and how they can be reduced upon a formal linearization to linear Korn inequalities on a surface, which play a fundamental role in the mathematical analysis of the linear Koiter shell model.

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Estimation Problem for Difference Descriptor System With Application to Network of Biosensors

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The purpose of this research is to offer constructive algorithm for estimator search in network model of biosensors. State variables and measurements are considered as random vectors. We use information cost criterion in order to find optimal estimator of inner product.

We apply general approach offered in [1] for modelling biosensor networks using information cost criterion. We use the following general descriptor system

$$x(t+1) = A(t)x(t) + u_x(t) + \xi(t), x(0) = x_0,$$

$$y(t) = C(t)x(t) + u_y(t) + \eta(t), t = \overline{0, T-1},$$

Assume sequences of random vectors $\{\xi(t), t = \overline{0, T-1}\}$, $\{\eta(t), t = \overline{0, T-1}\}$ are Gaussian noncorrelated between theirselves and between random vector x_0. $u_x(t)$, $u_y(t)$ are some disturbances affecting the system dynamics and corrupting the measurement vector correspondingly.

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An eco-epidemic competition model

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Eco-epidemic competition models have attracted increasing attention in the last years [1]. Empirical observations have shown that disease/parasites can affect in many different ways the outcome of species competition: as stronger competitor extinction or weaker competitor survival.

In this talk we present a competition eco-epidemic model that is rich enough to exhibit the behaviors above described and not so complext that it is analytically intractable. Competition is built up from the classical discrete time competition Leslie-Gower model [2] and the disease is introduced by means of a discrete SIS epidemic model with frequency-dependent transmission [3]. To our attention, this is the first discrete eco-epidemic competition model.

One of the differences between continuous and discrete models is that in the former all processes involved in the model (demography, competition and infection/recovery) occur together whereas in the later, it is usual to consider that processes take place sequentially [4]. Keeping this idea in mind, a key feature in the construction of this model is that a number k of epidemic-related process take place within each demographic/competition change. Making use of the existence of different time scales, it is possible to build up a reduced two dimensional system that approximates the behavior of the original system and that simplifies the analysis of the model [5].

The resulting reduced system generalizes in some sense the Leslie-Gower competition model and,for valuesof k large enough retains some of its properties, although the reduced model allows for disease-induced multistability scenarios.

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Some remarks on pointwise exponential dichotomy of skew-product flows

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In [2] Chow and Leiva introduced the notion of pointwise exponential dichotomy for skew-product flows. The first input-output criteria for pointwise exponential dichotomy was obtained by Megan, Sasu and Sasu in [3]. In this presentation, we expose a comparison between some pointwise and global criteria for exponential dichotomy of dynamical systems described by skew-product flows. We present several necessary and sufficient conditions for pointwise dichotomy and analyze their connections with previous results in this topic. Finally we discuss several future aims concerning the discrete pointwise dichotomy.

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Functional difference equations arising from renormalization in quasiperiodic systems

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The renormalization analysis of quasiperiodic dynamical systems (in which the dynamics are governed by an irrational number) leads to functional difference equations in which the spatial and temporal dynamics are linked, giving renormalization strange sets in function-pair space. In this talk we shall give an overview of the theory, of its application in areas such as quantum systems and strange non chaotic attractors, and of progress in the study of the associated functional difference equations.

Invariant measures of Markov operators associated to iterated function systems consisting of φ-max-contractions with probabilities

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We prove that the Markov operator associated to an iterated function system consisting of φ-max-contractions with probabilities has a unique invariant measure whose support is the attractor of the system.

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On Splitting with Different Growth Rates for Linear Discrete-Time Systems in Banach Spaces

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The aim of this work is to study some concepts of trisplitting with different growth rates for linear discrete-time systems in Banach spaces. Characterizations and connections between these concepts are given. As particular cases we obtain some results about different concepts of trichotomy (exponential, polynomial and with different growth rates).

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An efficient technique for solution of adsorption problems with steep moving profiles

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A numerical technique of orthogonal collocation on finite elements method using Hermite basis is applied to problems with steep gradients. The applicability of the method is shown for the solution of adsorption in solids with bidisperse pore structures. The results are shown in good agreement with the analytic ones when adsorption isotherm is linear. Comparison is made with the results of fitted mesh finite difference method and fitted collocation method. The technique is simple to apply and can be applied to widely applied to the models of adsorption and desorption in bidisperse solids with non linear isotherms.

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Stability Analysis and its Impact on the Parameters Estimation for a Logistic Growth Model

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We present a stability analysis of a system of differential equations describing the evolution of T-cells populations (see [2]). The analyzed system corresponds to a well-known four-compartmental model of the thymus which involves a logistic growth term. Unlike existing results on stability for this model which focus either only on the global population or on the two dominant double-positive and double-negative populations, the results presented in this paper provide sufficient conditions for asymptotic stability of all four populations, taken separately. The derived conditions involve parameters related to cells proliferation, death and transfer rates and are used as constraints in the least-square optimization procedure which provides estimates for all parameters of the model. The usage of the constraints derived from the stability analysis ensures that the estimated parameters lead to a mathematical model with an asymptotically stable fixed point.

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A Perron type approach for the nonuniform exponential dichotomy for evolution families

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The purpose of this paper is to highlight a sufficient condition in order to prove that an evolution family without any exponential growth is nonuniform exponentially dichotomic, by using a Perron type approach. The Perron problem for nonuniform dichotomy establishes the connection between the asymptotic behaviour of the homogeneous system and different outputs of the nonhomogeneous associated system. More precisely for each continuous and unbounded function on $\mathbb{R}_+$, if the nonhomogeneous system has an unbounded solution and the associated evolution family has an uniform exponential growth, then the homogeneous system is uniformly stable. Such a problem is frequently seen in the study of asymptotic uniform behaviour of nonautonomous systems.

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Dynamics of a tourism sustainability model with time delay

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A three-dimensional tourism sustainability model with delay is considered, based on the non-delayed version developed in [1, 2, 4]. The model takes into account the interaction of tourists, environmental resources and the invested capital. We assume that the environmental resources and capital stock at time t depend on the number of tourists in the past [3], which justifies the introduction of a time delay in the mathematical model. As a first step, the positivity of the solutions of the delay differential system is proved and the existence of positive equilibrium states is discussed. Next, the stability and existence of Hopf bifurcations are investigated in a neighborhood of a positive equilibrium, choosing the delay as bifurcation parameter. Numerical simulations are provided to illustrate the results.

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Analysis of Piecewise Affine Control for Linear Delay Difference Equations

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This talk focuses on the robustness of a specific class of control laws, namely the piecewise affine (PWA) state feedback function. More precisely, we are interested in closed-loop systems emerging from linear dynamical systems controlled via feedback channels in the presence of varying transmission delays by a PWA controller defined over a polyhedral partition of the state-space. We exploit the fact that the variable delays are inducing some particular model uncertainty. Our objective is to characterize the delay invariance margins: the collection of all possible values of the time-varying delays for which the positive invariance of the corresponding region is guaranteed with respect to the closed loop dynamics. These developments are proving to be useful for the analysis of different design methodologies and, in particular, for model–based predictive control approaches. The proposed delay margin describes for example the admissible transmission delays for explicit predictive control implementation. From a different perspective, the delay margin further characterizes the fragility of predictive control implemented via the on-line optimization and subject to variable computational time.

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Difference Approximations for Pricing Barrier Options

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Barrier options are financial derivative contracts that are activated or extinguished when the price of the underlying asset crosses a certain level. Most models for pricing barrier options assume continuous monitoring of the barrier. However in practice many (if not all) barrier options traded in markets are discretely monitored. There are two main difficulties in solving problems for discrete barrier options: I. When the barrier is discretely monitored, a numerical method may be used to value the option. However this method increases calculation time exponentially with the numbers of barrier. II. For problems pricing discrete barrier options, one may use the trinomial method, but it is less effective when the barrier is very close to the current asset price. In order to resolve these two problems, we construct a new class of numerical method which is based on efficient finite difference approximation for the temporal derivative associated with the nonlinear Black-Scholes partial differential equation modeling these barrier options. The derivatives in the asset directions are approximated using spline approximations methods. We show that the proposed method is unconditionally stable and provide very competitive results.

Stability of capillary waves on fluid sheets

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The linear stability of finite-amplitude capillary waves on inviscid sheets of fluid is investigated using conformal mapping techniques [2]. Symmetric and antisymmetric travelling waves discovered by Kinnersley [3] are perturbed with superharmonic or subharmonic perturbations and regions of instability are found. The instability results are corroborated by time integration of the fully nonlinear unsteady equations [1].

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Asymptotic Behaviours of Linear Discrete-Time Evolution Cocycles in Banach Spaces

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The purpose of this paper is to emphasize current developments in the stability theory. Due to the important role played in the study of stable, instable, and, respectively, central manifolds, the properties of exponential dichotomy and trichotomy for difference equations represent two domains with an impressive development. We intend to study a general model for nonautonomous linear discret-time dynamical systems in Banach spaces, the so called discrete-time evolution cocycles.

To this aim, we give some definitions and characterizations for the properties of exponential stability and instability, and we extend these techniques to obtain a unified study of the exponential dichotomy and trichotomy. While the classic theory of dichotomy deals with differential and difference equations with uniquely determined forward and backward solutions, nowadays applications require a corresponding theory for equations whose backward solutions are not guaranteed to exist or to be unique. To this goal, we define and characterize a more general dichotomic behaviour, the exponential splitting. We underline the results by several examples.

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Group analysis of differential-difference equations and conservation laws

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Symmetry method provides significant insights and techniques for finding exact analytic solutions of differential equations. These techniques have also been extended to discrete as well as semi-discrete equations during last decades. However, in literature, some inconsistency for group analysis of differential-difference equations (DDEs) was realised, which has been troublesome for quite some time. In this talk, the mystery of inconsistency is uncovered as the consequence of non-commutativity of (difference) shift operators and differential operators obtained by acting on the standard differential operators by group actions. Noether's theorem is extended to DDEs; computation of Noether's conservation laws of DDEs is illustrated with several examples.

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Peak Effects in Stable Linear Difference Equations

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Stability theory for linear difference equations is a mature discipline [1]; however, to the best of our knowledge, no attention has been paid to the non-asymptotic behavior of solutions for nonzero initial conditions. We show that solutions of stable linear difference equations may experience large deviations (peaks of trajectories) from initial conditions at finite time instants; this phenomenon is similar to that of differential equations [3]. Three results are presented in this paper.

First, lower bounds on the peak are obtained for nth order difference equations with equal real roots $\lambda_i \equiv \rho \in (\frac{1}{n}, 1)$ and initial conditions $(0,\ 0,\ ,\dots,1)$.

Second, we prove that for real roots $\lambda_i \geq \rho$, $i = 1,\dots,n$, the peak is always greater than the one observed for equal real roots $\lambda_i \equiv \rho$.

Third, we formulate the conditions on the coefficients a, b of the difference equation with characteristic polynomial $\lambda^{n+1} - a\lambda^n + b$ (see [2]) which unavoidably lead to peaks for specific initial conditions; lower bounds for the value of peak are provided.

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Transverse vibrations analysis of a beam with degrading hysteretic behavior by using Euller-Bernoulli beam model

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The paper is based on the analytical and experimental results from [3]-[4] and reveals, by numerical methods, the degradation of material stiffness due to the decrease of the first natural frequency, when the driving frequency is slightly lower than the first natural frequency of the undegradated structure. By considering the vibration of the uniform slender cantilever beam as an oscillating system with degrading hysteretic behavior the following equation is considered

$$\frac{\partial^2 y(x,t)}{\partial t^2} + 2\zeta(t)\omega(t)\frac{\partial y(x,t)}{\partial t} + l^4\omega^2(t)\frac{\partial^4 y(x,t)}{\partial x^4} = 0,$$

subjected to the boundary conditions

$$y(0,t) = y_0 \sin(\omega_{\text{input}} t),\ \frac{\partial y(0,t)}{\partial x} = 0,\ \frac{\partial^2 y(l,t)}{\partial x^2} = 0,\ \frac{\partial^3 y(l,t)}{\partial x^3} = 0.$$

To approximate the solution of the this problem, we use the method of Newton interpolating series (see [1]) and the Taylor series method (see [2]).

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Instability of a discrete reaction-diffusion equation

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Let $\mathcal{G} = (N, E)$ be a simple graph. A process of reaction and subsequent diffusion of two components (chemical, populations, ideologies etc.) on $\mathcal{G}$ can be described by the coupled recurrences

$$\begin{aligned} x_i(t+1) &= (1-d_1)f\big(x_i(t), y_i(t)\big) + \sum_{\{i,j\}\in E} \frac{d_1}{\sigma_j} f\big(x_j(t), y_j(t)\big), \\ y_i(t+1) &= (1-d_2)g\big(x_i(t), y_i(t)\big) + \sum_{\{i,j\}\in E} \frac{d_2}{\sigma_j} g\big(x_j(t), y_j(t)\big), \end{aligned} \qquad i = 1, 2, \dots, |N|. \tag{1}$$

Here x_i and y_i represent amounts of the components (concentration, abundance, intensity etc.), d_1 and d_2 denote dispersion probabilities of the respective components, σ_j denotes the degree of the j-th node, and functions f, g describe the reaction of the components in a node. We assume that the reaction system

$$u(t+1) = f\big(u(t), v(t)\big), \; v(t+1) = g\big(u(t), v(t)\big)$$

possesses an asymptotically stable equilibrium $(u^*, v^*) \in \mathbb{R}^2_+$. Consequently, $x_i(t) \equiv u^*$, $y_i(t) \equiv v^*$ is a spatially homogeneous equilibrium of the system (1). The aims of the contribution are to demonstrate that this equilibrium need not to be stable and to find conditions for its instability.

The research is motivated by attempts to model a diffusion dynamics of religious ideas and behavior forms in the ancient Mediterranean, see *www.gehir.phil.muni.cz.*

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On dynamics of triangular maps of the square with zero topological entropy

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It is known that, for interval maps, zero topological entropy is equivalent with bounded topological sequence entropy as well as with the non-existence of Li-Yorke scrambled triples. In this paper we answer the question how the situation changes when instead of interval maps triangular maps of the unit square are concerned.

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Nonlinear Fokker-Planck equations associated with weighted Tsallis and Kaniadakis entropies

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Nonlinear Fokker-Planck equations have important applications for modeling complex system in physics, engineering, biophysics, population dynamics, human movement sciences, neurophysics and economics. The phenomena described by this class of equations present a fundamental physical mechanism in common. The result of cooperative interactions between the subsystems of complex systems consists in the reduction of the number of degrees of freedom and self-organization of their subunits into synergetic entities. These synergetic systems admit low dimensional descriptions in terms of nonlinear Fokker-Planck equations, that describe the essential dynamics underlying the modeled phenomena. Recently, Wada and Scarfone [1] derived a non-linear Fokker-Planck equation for non-equilibrium systems related to the κ-entropy. In 2009, Wada and Scarfone [2] studied the asymptotic behavior of a nonlinear diusive equation obtained in the framework of the κ-generalized statistical mechanics. This kinetic equation can be characterized by the associated Lyapunov functional or Bregman type divergence, which is related to the difference of the κ-generalized free-energies. In standard linear Fokker-Planck equations, Lyapunov functionals are related with Kullback-Leibler divergences or relative entropies. Recently, for some nonlinear generalizations of Fokker-Planck equations, Lyapunov functionals were derived and discussed in relatively general context. The aim of this paper is to develop an integrated Fokker-Planck approach to quantum statistics, linear nonequilibrium thermodynamics and generalized extensive and nonextensive thermostatistics. In the context of generalized statistical mechanics, nonlinear Fokker-Planck equations associated with weighted Tsallis and Kaniadakis entropies

are constructed. Also, the properties and asymptotic behavior of complex systems described by these classes of equations are investigated. Our approach extends and generalizes recent results in this field.

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On a system of m difference equations having exponential terms

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In this work we obtain results concerning the behaviour of the positive solutions for the following cyclic system of difference equations [3]:

$$\begin{aligned} x^{(i)}_{n+1} &= a_i x^{(i+1)}_n + b_i x^{(i)}_{n-1} e^{-x^{(i+1)}_n}, \quad i = 1, 2, \ldots, m-1, \\ x^{(m)}_{n+1} &= a_m x^{(1)}_n + b_m x^{(m)}_{n-1} e^{-x^{(1)}_n}, \end{aligned} \tag{1}$$

$n = 0, 1, \ldots$, a_i, b_i, $i = 1, 2, \ldots, m$ are positive constants and the initial values $x^{(i)}_{-1}, x^{(i)}_0$, $i = 1, 2, \ldots, m$ are positive numbers. More precisely, we study the existence of the unique nonnegative equilibrium of (1). In addition, we investigate the boundedness and the persistence of the positive solutions of this system. Finally, we investigate the convergence of its positive solutions of to the unique nonnegative equilibrium.

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Discrete Neuron Model using piece-wise difference equations with a periodic internal decay rate with various periods

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We will discuss the patterns of periodic solutions, patterns of eventually periodic solutions and patterns of unbounded solutions of our Piece-wise Difference Equation that is used as a discrete neural model. In addition, we will compare the similarities and differences in the patterns between the periodic cycles and patterns of the transient terms when eventually periodic solutions exist. Furthermore, we will address the vital question: do eventually periodic solutions always exist or not?

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Exponential Stability in Volterra Difference Equations

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We use Lyapunov functionals to obtain sufficient conditions that guarantee exponential stability of the zero solution of the finite delay Volterra difference equation

$$x(t+1) = a(t)x(t) + \sum_{s=t-r}^{t-1} b(t,s)x(s).$$

Also, by displaying a slightly different Lyapunov functional we obtain conditions that guarantee the instability of the zero solution. The highlight of the paper is relaxing the condition $|a(t)| < 1$. Moreover we provide examples in which we show that our theorems provide an improvement of some of the recent literature.

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Numerical Modeling of Transmission Dynamics of Hepatitis C Virus among Injecting Drug Users (IDUs)

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Mathematical modeling of infectious diseases is a tool to understand the mechanism of how disease spreads and how it can be controlled. Numerical Modeling involves construction, implementation and analysis of reliable numerical schemes to solve continuous models. These schemes are constructed with the aim that the numerical model must preserve all the essential features of the continuous dynamical system. In this paper, the transmission dynamics of hepatitis C virus among the Injecting Drug Users (IDUs) has been analyzed numerically. A novel unconditionally stable Non-Standard Finite Difference (NSFD) numerical model is proposed and its convergence analysis is presented. Numerical experiments are performed and results are compared with standard finite difference schemes being already used to handle such problems. These schemes are conditionally convergent and may diverge for certain values of discretization parameter. The proposed numerical scheme is dynamically consistent with the biological nature of the continuous model and preserves all of its essential properties.

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Template iterations of quadratic maps and hybrid Mandelbrot sets

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The behavior of orbits for iterated polynomials has been widely studied since the dawn of discrete dynamics as a research field, in particular in the context of the complex quadratic family $f\colon \mathbb{C} \to \mathbb{C}$, parametrized as $f_c(z) = z^2 + c$, with $c \in \mathbb{C}$. While more recent research has been studying the orbit behavior when the map changes along with the iterations, many aspects of non-autonomous discrete dynamics remain largely unexplored.

Our work is focused on iterations of two quadratic maps $f_{c_0} = z^2 + c_0$ and $f_{c_1} = z^2 + c_1$, according to a prescribed binary sequence, which we call template [1]. We study the asymptotic behavior of the critical orbit, and define the Mandelbrot set in this case as the locus for which this orbit is bounded. Unlike in the case of single maps, this concept can be understood in several ways. For a fixed template, one may consider the subset of the parameter space of (c_0, c_1) in $\mathbb{C}^2$ for which the iteration is critically bounded. Alternately, one may consider, for fixed complex parameters, the subset of templates which lead to a bounded critical orbit [2]. We approach both types of sets, as well as hybrid combinations of them, we investigate their basic topological and fractal properties and propose applications to epigenetics.

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Decoupling and simplifying dynamical systems on time scales

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We consider the dynamic system in a Banach space on unbounded above time scale

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,y), \\ y^{\Delta} = B(t)y + g(t,x,y). \end{cases} \tag{1}$$

This system satisfies the conditions of integral separation with the separation constant ν, nonlinear terms are ε-Lipshitz, and the system has a trivial solution. We find sufficient conditions under which the system (1) is locally dynamic equivalent

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,u(t,x)), \\ y^{\Delta} = B(t)y + g(t,\kappa(t,x,y),y). \end{cases} \tag{2}$$

If (1) is regressive and time scale is unbounded below, than the system (1) is locally dynamic equivalent

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,u(t,x)), \\ y^{\Delta} = B(t)y + g(t,v(t,y),y). \end{cases} \tag{3}$$

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Discrete Approximations to Volterra Equations

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The asymptotic behaviour of solutions to linear Volterra integral equations of the form

$$x(t) = \int_0^t k(t,s)x(s)\,ds + f(t), \tag{$\star$}$$

for t in $[0,\infty)$, has been extensively studied. Sharp conditions on the kernel k and forcing function f have been found [1] to ensure that $x(t)$ converges to a constant c as $t \to \infty$.

There is considerable literature [2, 3] on the approximation of solutions of ($\star$) on compact intervals $[0,T]$. This talk examines the problem as to how the asymptotically constant solutions on $[0,\infty)$ should be approximated numerically. The approach is to impose natural conditions on the kernel k, the forcing function f and the quadrature rule used, and show that these imply that the discretisation has suitable properties. The method seems to fail when c is not explicitly known, as can happen.

There is considerable literature [2, 3] on the approximation of solutions of ($\star$) on compact intervals $[0,T]$.

This is joint work with E. Messina (The University of Naples Federico II) and A. Vecchio (CNR - Naples)

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Dichotomy spectrum and topological conjugacy on nonautonomous unbounded difference system

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We will consider the nonautonomous linear system

$$x(n+1) = A(n)x(n) \tag{1}$$

where $x(n)$ is a column vector of $\mathbb{R}^d$ and the matrix function $n \mapsto A(n) \in \mathbb{R}^{d\times d}$ is non singular. We also assume that (1) has an exponential dichotomy on $\mathbb{Z}$ [1] with projector $P = I$. We also consider the perturbed system

$$w(n+1) = A(n)w(n) + f(n, w(n)) \tag{2}$$

where $f\colon \mathbb{Z} \times \mathbb{R}^d \to \mathbb{R}^d$ is continuous in $\mathbb{R}^d$ is a Lipschitz function such that $n \mapsto f(n, 0)$ is bounded for any $\mathbb{Z}$.

We will present a result with sufficient conditions ensuring that (1) and (2) are topologically equivalent, namely the existence of a map $H\colon \mathbb{Z} \times \mathbb{R}^d \to \mathbb{R}^d$ with the following properties: i) For each fixed $n \in \mathbb{Z}$, the map $u \mapsto H(n, u)$ is a bijection. ii) For any fixed $n \in \mathbb{Z}$, the maps $u \mapsto H(n, u)$ and $u \mapsto H^{-1}(n, u) = G(n, u)$ are continuous. iii) If $x(n)$ is a solution of (1), then $H[n, x(n)]$ is a solution of (2). Similarly, if $w(n)$ is a solution of (2), then $G[n, w(n)]$ is a solution of (1).

This result can also be seen as a generalization of a continuous result obtained by Lin in [3].

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Stochastic difference equations with the Allee effect

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For a truncated stochastically perturbed equation $x_{n+1} = \max\{f(x_n) + l\chi_{n+1}, 0\}$ with $f(x) < x$ on $(0, m)$, which corresponds to the Allee effect, we observe that for very small perturbation amplitude l, the eventual behavior is similar to a non-perturbed case: there is extinction for small initial values in $(0, m-\varepsilon)$ and persistence for $x_0 \in (m+\delta, H]$ for some H satisfying $H > f(H) > m$. As the amplitude grows, an interval $(m-\varepsilon, m+\delta)$ of initial values arises and expands, such that with a certain probability, x_n sustains in $[m, H]$, and possibly eventually gets into the interval $(0, m-\varepsilon)$, with a positive probability. Lower estimates for these probabilities are presented. If H is large enough, as the amplitude of perturbations grows, the Allee effect disappears: a solution persists for any positive initial value.

The approximation of boundary controls for the one-dimensional wave equation

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We consider a finite-differences semi-discrete scheme for the approximation of boundary controls for the one-dimensional wave equation. The high frequency numerical spurious oscillations lead to a loss of the uniform (with respect to the mesh-size) controllability property of the semi-discrete model in the natural setting. We prove that, by filtering the high frequencies of the initial data in an optimal range, we restore the uniform controllability property. Moreover, we obtain a relation between the range of filtration and the minimal time of control needed to ensure the uniform controllability, recovering in many cases the usual minimal time to control the (continuous) wave equation.

More precisely, we improve the results of [1] by filtering in an optimal way the initial condition. We obtain a precise estimate on the minimal time needed that turns out to be optimal as soon as we filter enough frequencies. We emphasize that beyond the theoretical interest of our result, it is likely that it is of interest to try to allow filtrations which contain as many modes as possible, in order to improve the precision of the approximation.

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Iterated Means Dichotomy for Discrete Dynamical Systems[2]

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In this paper, we discuss the dichotomy of iterated means of discrete dynamical systems acting on a compact convex subset of the finite dimensional space. As an application, we study the mean ergodicity of non-homogeneous Markov chains.

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[2]This work has been partially supported by the MOHE grant FRGS14-141-0382.

On the asymptotic stability of discrete crocodilians model

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The sex ratio of crocodiles is strongly biased towards females, often as high as 8 females to 1 male. In crocodilians, the temperature of egg incubation is the environmental factor determining sex. If the temperature is low, around 30, the hatchlings are all females. Highter temperature, around 34, hatch all males.

This study was made to consider the asymptotic stability of a positive equilibrium point in a nonlinear discrete model of the basic nesting population model, which is described in three-region depending on the temperature of egg incubation. This basic model based on key life-history data [5] and Murray's research [3, 6]. To study above, we have applying the classical linearization method [1, 2] and a luxury Liapunov function [4].

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A remark on difference equations

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We rephrase simple observations on difference equations, including the Z-transform, by adopting an operator-theoretic perspective. This is joint work with Rainer Picard and Sascha Trostorff.

Approximation of the Solution of Stochastic Differential Equations Driven by Step Fractional Brownian Motion

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The aim of this paper is to approximate the solution of a stochastic differential equation driven by step-fractional Brownian motion using a series expansion for the noise. We prove that the solution of the approximating equations converge in probability to the solution of the given equation.

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Snap-back repellers and critical homoclinic orbits

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A nondegenerate homoclinic orbits to an expanding fixed point of a map $F : X \to X$, $X \subseteq \mathbb{R}^n$ is called a *snap-back repeller*. It is known that the existence of a snap-back repeller (in its original definition) implies the existence of an invariant set on which the map is chaotic [4, 1, 2, 5]. However, it not well known when the first homoclinic orbit appears, and when other homoclinic explosions, i.e., appearance of infinitely many new homoclinic orbits, occur. Our aim is to characterize these bifurcations, for any kind of maps, smooth or piecewise smooth, continuous or discontinuous, defined in a bounded or unbounded set. For this we define a *noncritical homoclinic orbit* [3], then a homoclinic orbit of an expanding fixed point is structurally stable iff it is noncritical, that is, only critical homoclinic orbits are responsible for the homoclinic explosions. Different kinds of critical homoclinic orbits are investigated, as well as their role for the dynamics of the map.

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Construction of the first integrals for two classes of ordinary difference equations

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In [1] we presented a broad class of the systems of ordinary difference equations (OΔE) sharing the property to have a nontrivial Lax representation. This representation, in principle, gives a simple procedure to derive some number of the first integral for the system under consideration but this can be done only for this concrete system but not for whole class which involve this system. In our report we set and discuss the problem of constructing the first integrals for whole infinite classes of OΔE given in [1].

In a report we consider two infinite classes of ordinary difference equations of the form

$$T(i+k+s)\tilde{T}_s^k(i) = T(i)\tilde{T}_s^k(i+2) \tag{1}$$

and

$$T(i+k)\tilde{S}_s^k(i) = T(i+s)\tilde{S}_s^k(i+2) \tag{2}$$

for $k \geq 1$ and $s \geq k+1$, where the discrete polynomials $\tilde{T}_s^k(i)$ and $\tilde{S}_s^k(i)$ are defined in [1] and [2]. All these equations, in fact, can be rewritten in normal form

$$T(i+k+s) = R(T(i), \ldots, T(i+k+s-1))$$

with some rational function R which depends also on some constants $(c_1, \ldots, c_k)$. Based on actual calculations using the Lax representation, we propose an approach to construct the first integrals for the OΔE (1) and (2). By and large this approach does not depend on the Lax representation and allows to construct a number of integrals for whole classes equations (1) and (2) in terms of special classes of discrete polynomials.

It is worthwhile to distinguish two cases: $k+s = 2g+1$ and $k+s = 2g+2$. With some technical propositions, we derive g the first integrals in the first case and $g+1$ the first integrals in the second case. All these integrals also appear as the coefficients of equation $P(z,w) = 0$ of hyper-elliptic spectral curve. We prove that solution spaces $\mathcal{N}_s^k$ with fixed value of $k+s$ are organized in chains of inclusions.

These results suggest that these ordinary difference equations may be integrable in a Liouville-Arnold sense.

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Fisher-Kolmogorov type perturbations of the relativistic operator: differential vs difference

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We are concerned with existence of multiple periodic solutions for differential equations involving Fisher-Kolmogorov perturbations of the relativistic operator of the form

$$-\left[\phi(u')\right]' = \lambda u(1 - |u|^q)$$

as well as for difference equations, of type

$$-\Delta\left[\phi(\Delta u(n-1))\right] = \lambda u(n)(1 - |u(n)|^q);$$

here $q \in (1, \infty)$ is fixed, $\Delta u(n) = u(n+1) - u(n)$ is the forward difference operator, $\lambda > 0$ is a real parameter and

$$\phi(y) = \frac{y}{\sqrt{1-y^2}} \quad (y \in (-1, 1)).$$

The approach is variational and relies on a generalization of a result for smooth functionals of Clark [1] to convex, lower semicontinuous perturbations of C^1- functionals due to Szulkin [3]. This talk is based on joint work with Petru Jebelean.

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On the Spectrum of dynamical systems on trees

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In their paper, Schweizer and Smítal [1] introduced the notions of distributional chaos for continuous maps of the interval, spectrum and weak spectrum of a dynamical system. Among other, they have proved that in the case of continuous interval maps, both the spectrum and the weak spectrum are finite and generated by points from the basic sets. Here we generalize the mentioned results for the case of continuous maps of a finite tree. While the results are similar, the original argument is not applicable directly and needs essential modifications. In particular, it was necessary to resolve the problem of intersection of basic sets, which was a crucial point.

An example of one-dimensional dynamical system with an infinite spectrum is presented.

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On a Family of First-Order Piecewise Linear Systems

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We will give a detailed analysis complete with open problems and conjectures of the global character of the solutions of the piecewise linear difference equations $x_{n+1} = |x_n| + ay_n + b$ and $y_{n+1} = x_n + c|y_n| + d$ where $a, b, c, d \in \{-1, 0, 1\}$ and $(x_0, y_0) \in R^2$ with emphasis on the special cases that exhibit unbounded or periodic solutions.

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Lyapunov type inequalities for forced sub and super-half-linear dynamic equations on time scales

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In this paper, we present some new Lyapunov type inequalities for second-order forced dynamic equations on time scales $\mathbb{T}$ of the form

$$\left(r(t)\Phi_\beta(x^\Delta(t))\right)^\Delta + q(t)\Phi_\gamma\left(x^\sigma(t)\right) = f(t); \qquad t \in [t_0, \infty)_\mathbb{T}$$

in the sub-half-linear ($0 < \gamma < \beta$) and the super-half-linear ($0 < \beta < \gamma < 2\beta$) cases where $\Phi_*(s) = |s|^{*-1}s$. No sign restrictions are imposed on the potential q, and the forcing term f.

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On AGIIFSs having attractor

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We study affine generalized infinite iterated function systems (for short AGIIFSs). One of the main tools that we use in our work is a technique introduced by F. Strobin and J. Swaczyna which allows to associate to each $n \in \mathbb{N}^*$ and each AGIFS $\mathcal{F}$ a new AGIFS $\mathcal{F}_n$. Our main result states that the following statements are equivalent: a) There exists $n \in \mathbb{N}^*$ such that $\mathcal{F}_n$ is hyperbolic. b) There exist $n \in \mathbb{N}^*$ and a comparison function φ such that $\mathcal{F}_n$ is φ-hyperbolic. c) There exists $n \in \mathbb{N}^*$ such that $\mathcal{F}_n$ has attractor. d) $\mathcal{F}$ has attractor. e) There exists $n \in \mathbb{N}^*$ such that $\mathcal{F}_n$ is strictly topologically contractive.

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Solution spaces of homogeneous linear difference systems with coefficient matrices from commutative groups

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The talk is based on the joint works [1, 2] with Petr Hasil. We analyse the solution spaces of limit periodic homogeneous linear difference systems, where the coefficient matrices of the considered systems are taken from a commutative group which does not need to be bounded. In particular, we study such systems whose fundamental matrices are not asymptotically almost periodic or which have solutions vanishing at infinity. We identify a simple condition on the matrix group which guarantees that the studied systems form a dense subset in the space of all considered systems. The obtained results improve previously known theorems about non-almost periodic and non-asymptotically almost periodic solutions. Note that the elements of the coefficient matrices are taken from an infinite field with an absolute value and that the corresponding almost periodic case is treated as well.

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On Bailey's fixed point theorem in fuzzy metric spaces

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We present a general fixed point result for mappings with contractive iterates in compact fuzzy metric spaces of George and Veeramani type, extending theorems in [1] and [2]. Motivated by [3], we suggest an application for the convergence analysis of a particle swarm optimization algorithm.

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Acknowlegdements

We would like to express our special thanks and gratitude for the financial support offered by:

- ◇ West University of Timişoara
- ◇ Faculty of Mathematics and Computer Science
- ◇ West University of Timişoara Foundation[1]
- ◇ Consiliul Judeţean Timiş[2]
- ◇ Netex Consulting

We acknowledge with very special thanks the support offered by the Rector's Chancellery in order to obtain special facilities for the ICDEA participants.

Our warm thanks go to Department of Communication, Image and Institutional Marketing for their constant support in realizing the graphics of the conference's materials.

We are grateful to Department of Information Systems and Digital Communications for our collaboration at the creation and at the permanent updating of the conference's website.

We would like to thank Tourist Information Centre Timişoara (Timişoara City Hall) for the maps of Timişoara offered for the participants. For more info please visit: http://www.timisoara-info.ro/

[1]Fundaţia Universitatea de Vest din Timişoara (in Romanian)
[2]Acţiune cofinanţată de Consiliul Judeţean Timiş.

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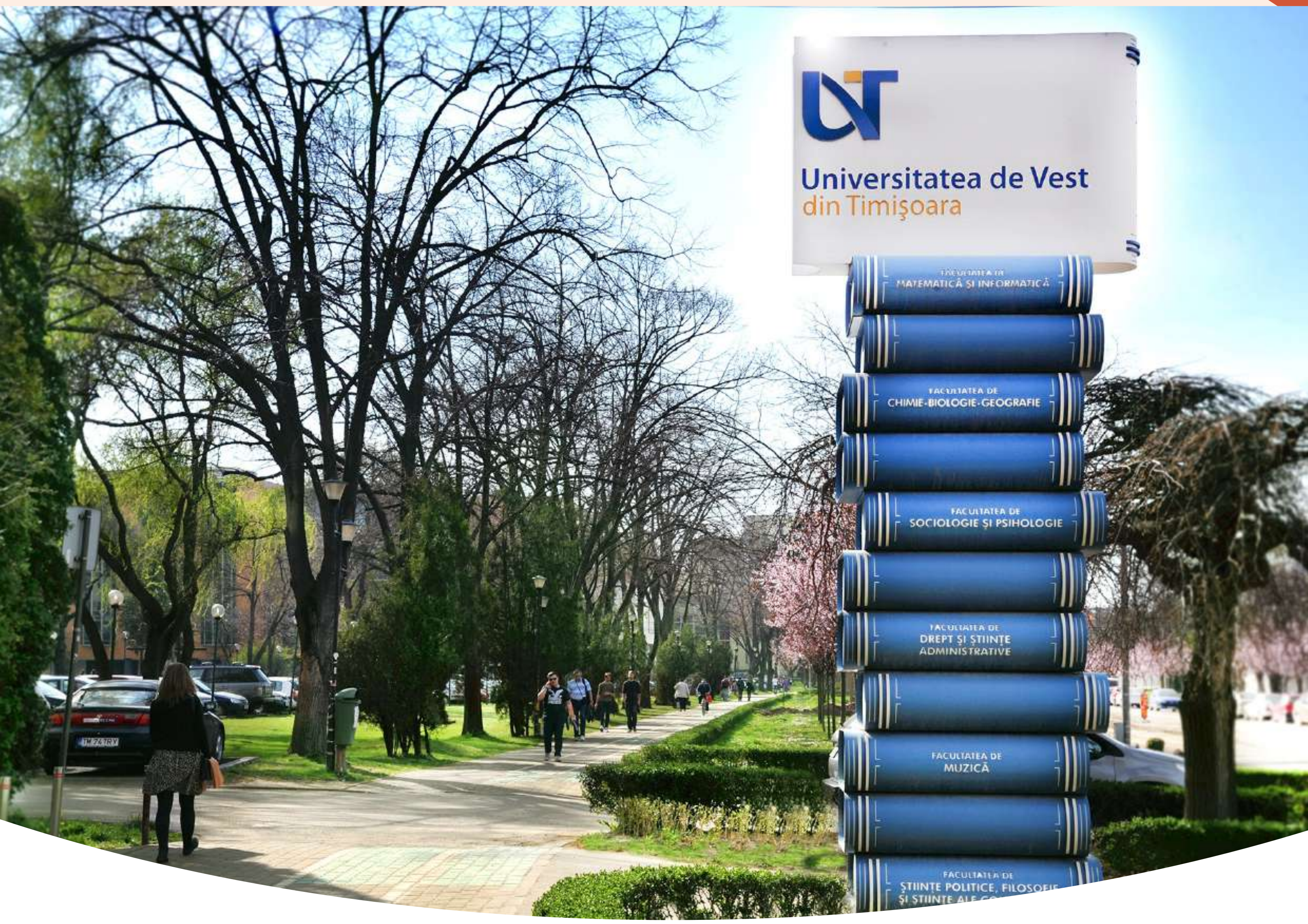

Special thanks for the financial support offered by

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